Cubic-phase-free dispersion compensation in solid-state ultrashort-pulse lasers

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We show that intracavity group-velocity dispersion compensation with the use of prisms composed of conventional optical materials can be accomplished while simultaneously eliminating the round-trip cavity cubic phase. The ability to compensate perfectly both second- and third-order dispersion exists for pulses whose central wavelengths lie within a range that depends on the prism and laser rod materials as well as on the prism angles. In the case of Ti:sapphire and Cr:LiSrAlF$_6$ lasers, Brewster prisms composed of readily available materials can be used to compensate for both group-velocity dispersion and cubic phase over much of the respective tuning ranges.

Recently there has been much progress in generating laser pulses of less than 50 fs in duration from mode-locked Ti:sapphire lasers.[1-9] In addition, pulses as short as 33 fs have been reported from a mode-locked Cr:LiSrAlF$_6$ (Cr:LiSAF) laser.[10] In each case, the major factor believed to limit the pulse duration is residual cavity cubic phase $(d^3\phi/d\omega^3)$. The effect of cavity cubic phase on the generated pulse duration has been considered theoretically by Brabec *et al.*[11] Techniques that have been used to reduce the amount of cavity cubic phase include reducing the length of the laser rod,[2,6] use of nonstandard prism materials,[1-3] and use of an intracavity Gires–Tournois interferometer.[4] All these techniques have resulted in reduced cavity cubic phase and shorter pulses. In this Letter we show that, for a wide range of operating wavelengths, one can eliminate cavity cubic phase altogether while still compensating for group-velocity dispersion (GVD) of the laser medium. This is accomplished simply by correct choice of prism material, prism separation, and intraprism path length. Our analysis suggests the possibility of generating even shorter pulses than have been produced to date, with pulse durations being limited only by fourth-order dispersion and the gain bandwidth of the laser medium.

Calculations of prism-pair group-delay dispersion (GDD = $d^2\phi/d\omega^2$) and cubic phase most often use the formalism presented in Refs. 12–14. This formalism, however, becomes complicated when one wishes to consider prisms operated at other than minimum deviation and Brewster's angle of incidence.[15] In addition, formulas for higher orders of dispersion become quite complex. For these reasons, we have chosen to use dispersive ray tracing for all the calculations in this Letter. Angles are calculated from Snell's law, and the index of refraction is calculated from a dispersion equation. Intraprism and interprism ray path lengths are calculated, and the total round-trip phase is computed for several wavelengths. Any order of dispersion, within the precision limits of the computer, can then be quickly

calculated by numerical differentiation, and the results agree exactly with the formulas of Refs. 12–14 when applicable.[16] These calculations, as do those of Refs. 12–15, assume a collimated beam.

We have previously reported[1] that a cavity consisting of a Ti:sapphire rod and two BeO prisms can simultaneously have zero GDD and zero cubic phase at 800 nm if the prism separation and intraprism path length are correctly chosen. BeO is the only material that we have found that exhibits this behavior near the peak of the gain profile of Ti:sapphire. However, at longer wavelengths within the gain profile of Ti:sapphire, many other prism materials also exhibit this behavior. The often-made assumption that cavity cubic phase is minimized when intraprism material is minimized is incorrect for prisms made of these materials at these wavelengths.

To understand this phenomenon, let us first examine the dispersive behavior of a cavity consisting of a 1-cm Ti:sapphire rod and two Brewster prisms in the limit of zero intraprism material path length. For the time being, assume that the prism separation is always set to exactly cancel the positive GDD of the rod and thus give a net cavity GDD of zero. We may now consider the net cavity cubic phase as a function of wavelength. The cubic phase of the rod is positive and increases with increasing wavelength. The cubic phase of a prism pair composed of any typical optical material is negative in the visible range and increases with increasing wavelength, crossing zero between 900 and 1300 nm. Thus, for a given prism material, there will be some wavelength where the prism-pair cubic phase is equal and opposite to the cubic phase of the rod, resulting in a net cavity cubic phase of zero. We will call this wavelength λ_{0p}. In Fig. 1 we show as a function of wavelength the magnitude of the cubic phase for a 1-cm sapphire rod and for prism pairs composed of eight optical materials. The wavelength at which the prism and rod curves intersect is λ_{0p}. Because the dispersion of a prism pair in the limit of zero intraprism path is proportional to prism separation, the value of λ_{0p} is
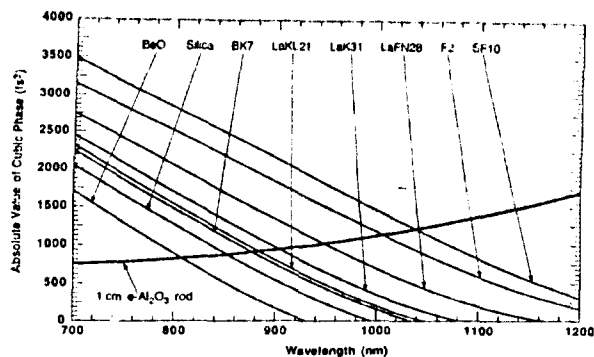
Fig. 1. Magnitude of round-trip cubic phase for a 1-cm e-sapphire rod and various Brewster prism pairs. Zero round-trip cavity GDD and zero intraprism path length are assumed.
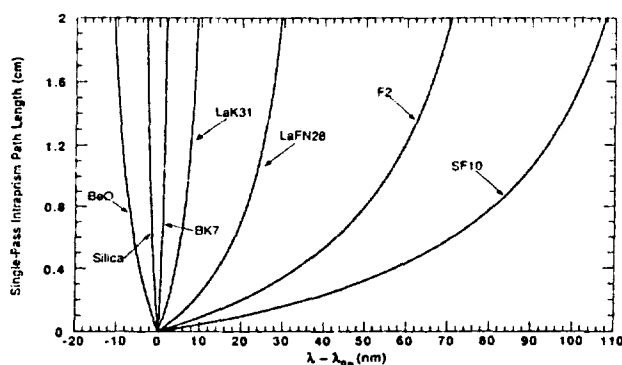

Fig. 2. Intraprism path length as a function of wavelength, relative to λ_{0p}, required to null simultaneously both GDD and cubic phase in a cavity consisting of a 1-cm Ti:sapphire rod and a Brewster prism pair.

independent of the length of the laser rod. The values of λ_{0p} range from 807 nm for BeO and 852 nm for silica all the way to 1038 nm for SF10. It should be noted that sub-100-fs pulse generation in Ti:sapphire has been demonstrated throughout this wavelength range, with pulses as short as 62 fs having been reported at 1053 nm.[6]

Now that we have considered the zero intraprism path-length limit, let us examine the effect of a finite path in the prism. Let l_m be the total single-pass intraprism path length at the central optical frequency. It can be shown that a cavity containing only a prism pair is dispersively equivalent to a similar cavity containing an identical prism pair with zero intraprism path length and a rod of length l_m composed of the prism material. This latter cavity is just the zero intraprism path cavity considered above, with the Ti:sapphire rod now replaced by a rod composed of the prism material. The zero cubic phase condition will now occur at the wavelength where the bulk cubic phase of the rod composed of prism material is equal and opposite to the cubic phase owing to the angular dispersion of the prism pair. Because this wavelength is independent of the rod length, one can achieve zero cubic phase for arbitrarily large prism material path lengths. We call this wavelength $\lambda_{\infty p}$, because it corresponds to the zero-cubic-phase wavelength of a laser cavity in the limiting case in which the intraprism path length is much greater than the length of the gain medium.

From the preceding arguments, it should now be clear that, to the extent that the prism material is different from the laser rod, λ_{0p} and $\lambda_{\infty p}$ define the end points of a range of wavelengths over which one can zero the cavity cubic phase by introducing the correct amount of intraprism path length. Figure 2 shows the single-pass intraprism path length required to achieve zero cubic phase and zero GDD as a function of wavelength. In practice, finite beam diameter and pulse bandwidth will impose a minimum on l_m, and cavity considerations will impose a maximum on the prism separation and hence on l_m. As a practical example, a prism pair with $l_m = 1$ cm will allow a 1-mm beam to have a bandwidth in excess of 1 rad/fs and still maintain a modest prism separation. These results are summarized in Table 1.

The data presented thus far have assumed Brewster-angle minimum deviation prisms and zero round-trip cavity GDD. Figure 3 shows the effect of nonzero round-trip GDD on λ_{0p}. As we see, negative

Table 1. Zero-Cubic-Phase Wavelength Ranges for Several Combinations of Brewster Prism and Laser Rod Materials[a]

Prism Material	Laser Rod Material	λ_{0p} (nm)	$\lambda_{\infty p}$ (nm)	Separation (cm) $l_m = 0$ cm	Separation (cm) $l_m = 1$ cm	Quartic Phase (fs⁴) $l_m = 0$ cm
BeO	Ti:sapphire	807	791	23.4	43.6	−3190
Silica	Ti:sapphire	852	847	48.1	79.7	−3340
BK7	Ti:sapphire	877	880	35.6	64.2	−3450
LaKL21	Ti:sapphire	884	887	20.2	42.1	−3540
LaK31	Ti:sapphire	904	917	15.7	35.9	−3630
LaFN28	Ti:sapphire	948	988	10.8	29.3	−3850
F2	Ti:sapphire	1006	1104	11.4	33.5	−4180
SF10	Ti:sapphire	1038	1178	5.8	23.6	−4630
Silica	Cr:LiSAF	702	847	19.5	47.0	−380
BK7	Cr:LiSAF	724	880	13.3	39.0	−500
LaK31	Cr:LiSAF	746	917	5.5	24.3	−630
LaFN28	Cr:LiSAF	775	988	3.4	20.8	−770
F2	Cr:LiSAF	809	1104	3.1	23.8	−930

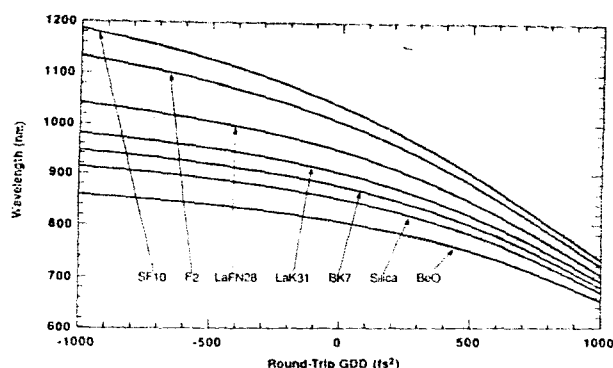[a] A 1-cm rod, zero round-trip GDD, and zero cubic phase are assumed.

Fig. 3. Zero-cubic-phase wavelength as a function of round-trip cavity GDD, assuming Brewster prisms, zero intraprism path length, and a 1-cm Ti:sapphire rod.
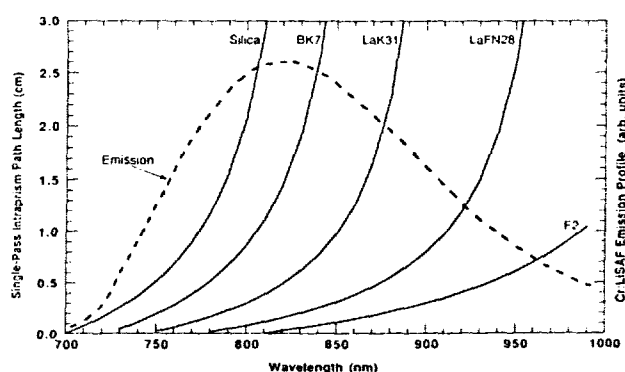

Fig. 4. Intraprism path length required to null cubic phase as a function of wavelength, assuming Brewster prisms, zero round-trip cavity GDD, and a 2-cm Cr:LiSAF rod. The dashed curve is the emission profile of Cr:LiSAF.

values of round-trip GDD shift λ_{0p} upward, whereas positive round-trip GDD shifts λ_{0p} downward. As a function of prism exit angle θ, λ_{0p} increases slowly until approximately 75°, where it begins to increase rapidly, going to infinity at $\theta = 90°$.

We have shown that there exist useful ranges of wavelengths over which a Ti:sapphire laser cavity can be made to have zero cubic phase. However, for $\lambda < 847$ nm, where the shortest pulses to date have been generated, we have found no commercially available prism material that will give zero cubic phase. This situation is quite different for Cr:LiSAF. As we can see from Fig. 4, zero cubic phase can be easily obtained at any wavelength within the tuning range of Cr:LiSAF. In particular, at 825 nm, BK7 Brewster prisms with 2 cm of single-pass intraprism path length will give zero cavity cubic phase. In general, less dispersive laser materials such as Cr:LiSAF will result in shorter zero-cubic-phase wavelengths λ_{0p}.

In this Letter we have presented calculations that clearly show that it is possible to design ultrashort-pulse solid-state laser cavities that have zero net cavity cubic phase. In particular, we have given examples of prism materials that may be used over much of the Ti:sapphire tuning range and over all the Cr:LiSAF tuning range to null simultaneously both

GDD and cubic phase. Many real cavity designs contain additional dispersive cavity elements, such as an acousto-optic modulator or dispersive multilayer dielectric mirrors. The results presented here hold for such cavities, although the zero-cubic-phase wavelengths will be shifted either up or down, depending on the sign of the mirror cubic phase and the composition of the acousto-optic modulator. We expect that zero-cubic-phase cavities of the type discussed here may lead to the production of pulses as short as or shorter than 10 fs.

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16. Note that the formula for $d^3 P/d\lambda^3$ given in Refs. 12–14 is an approximation. This derivative is used in the calculation of cubic phase. In order for it to agree exactly with our ray-tracing analysis of Brewster prisms it is necessary to use the exact form, which is

$$\frac{d^3 P}{d\lambda^3} = l \cos\beta \left[\left(\frac{24}{n^3} - 48n \right) \left(\frac{dn}{d\lambda} \right)^3 - 24 \frac{dn}{d\lambda} \frac{d^2 n}{d\lambda^2} \right]$$
$$+ l \sin\beta \left[\left(\frac{dn}{d\lambda} \right)^3 \left(\frac{12}{n^6} + \frac{12}{n^4} + \frac{8}{n^3} - \frac{16}{n^2} + 32n \right) \right.$$
$$\left. + \left(24n - \frac{12}{n^3} \right) \frac{dn}{d\lambda} \frac{d^2 n}{d\lambda^2} + 4 \frac{d^3 n}{d\lambda^3} \right].$$